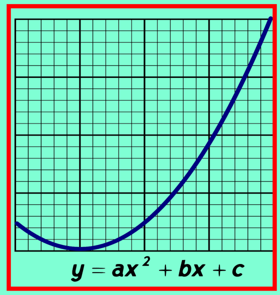
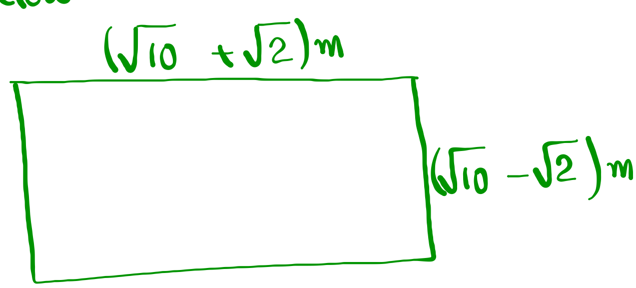
Math 125

Spring 2021

Lecture 19



Find the area and the perimeter of the shape below



Rectangle

$A = LW \Rightarrow m^2$

$P = 2L + 2W \Rightarrow m$

$A = LW = (\sqrt{10} + \sqrt{2})(\sqrt{10} - \sqrt{2}) = \sqrt{100} \cancel{- \sqrt{20}} \cancel{+ \sqrt{20}} - \sqrt{4}$

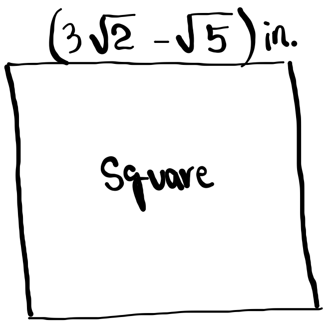
$= 10 - 2 = \boxed{8\ m^2}$

$P = 2L + 2W = 2(\sqrt{10} + \sqrt{2}) + 2(\sqrt{10} - \sqrt{2})$

$= 2\sqrt{10} \cancel{+ 2\sqrt{2}} + 2\sqrt{10} \cancel{- 2\sqrt{2}} = \boxed{4\sqrt{10}\ m}$

March 11, 2021

Find the area and the perimeter of the shape below:



Square

$A = S^2$ $\qquad$ $P = 4S$

in.^{2} $\qquad$ in.

$A = S^2 = (3\sqrt{2}-\sqrt{5})^2$

$= (3\sqrt{2}-\sqrt{5})(3\sqrt{2}-\sqrt{5})$

$= 9\sqrt{4} - 3\sqrt{10} - 3\sqrt{10} + \sqrt{25}$

$= 9\cdot 2 - 6\sqrt{10} + 5$

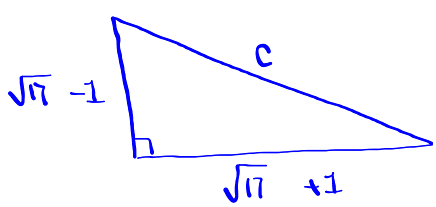
$= \boxed{23 - 6\sqrt{10} \text{ in}^2}$

$P = 4S$

$= 4(3\sqrt{2}-\sqrt{5})$

$= \boxed{12\sqrt{2} - 4\sqrt{5} \text{ in.}}$

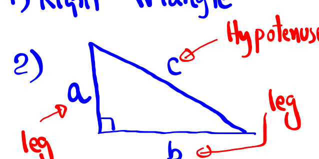
Consider the shape below



Find c.

1) Right Triangle

2)



$a^2+b^2=c^2$

Pythagorean Formula

$c^2 = (\sqrt{17}-1)^2 + (\sqrt{17}+1)^2$

$= (\sqrt{17}-1)(\sqrt{17}-1) + (\sqrt{17}+1)(\sqrt{17}+1)$

$= (\sqrt{17})^2 - \cancel{\sqrt{17}} - \cancel{\sqrt{17}} + 1 + (\sqrt{17})^2 + \cancel{\sqrt{17}} + \cancel{\sqrt{17}} + 1$

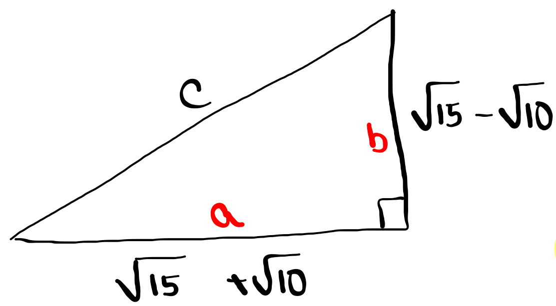
$= 17 + 1 + 17 + 1$

$c^2 = 36 \Rightarrow c = \sqrt{36}$ $\qquad$ $\boxed{c=6}$

Recall $x \geq 0$

$(\sqrt[n]{x})^n = x$

Find C



Right Triangle

Pythagorean Formula

$$a^2+b^2=c^2$$

$$(\sqrt{15}+\sqrt{10})^2+(\sqrt{15}-\sqrt{10})^2=c^2$$

$$(\sqrt{15}+\sqrt{10})(\sqrt{15}+\sqrt{10})+(\sqrt{15}-\sqrt{10})(\sqrt{15}-\sqrt{10})=c^2$$

$$(\sqrt{15})^2+\cancel{\sqrt{150}}+\cancel{\sqrt{150}}+(\sqrt{10})^2+(\sqrt{15})^2-\cancel{\sqrt{150}}-\cancel{\sqrt{150}}+(\sqrt{10})^2=c^2$$

$$15+10+15+10=c^2 \qquad c^2=50 \qquad c=\sqrt{50}$$

$$c=\sqrt{25}\sqrt{2} \qquad \boxed{c=5\sqrt{2}}$$

Solve

$$\sqrt{2x-7}+3=6$$

Isolate the radical

$$\sqrt{2x-7}=6-3$$

$$\sqrt{2x-7}=3$$

Raise both Sides to the index Power. Index=2

$$(\sqrt{2x-7})^2=3^2$$

$$2x-7=9 \qquad 2x=16$$

$$x=8 \checkmark$$

Check

$$\sqrt{2x-7}+3=6$$

$$\sqrt{2(8)-7}+3=6$$

$$\sqrt{16-7}+3=6$$

$$\sqrt{9}+3=6$$

$$3+3=6$$

$$6=6\checkmark$$

$$\{8\}$$

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Solve & check

$x - \sqrt{3x+10} = 0$

Always isolate the radical first.

$x = \sqrt{3x+10}$

Square both sides

$(x)^2 = \left(\sqrt{3x+10}\right)^2$

$x^2 = 3x + 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x - 5 = 0$ $\quad$ $x + 2 = 0$

✓ $\boxed{x = 5}$ $\quad$ $\cancel{x = -2}$

E.S.

$\{5\}$

check

$x - \sqrt{3x+10} = 0$

$x = 5$ ✓

$5 - \sqrt{3(5)+10} = 0$

$5 - \sqrt{25} = 0$

$5 - 5 = 0$ ✓

$x = -2$

$-2 - \sqrt{3(-2)+10} = 0$

$-2 - \sqrt{-6+10} = 0$

$-2 - \sqrt{4} = 0$

$-2 - 2 = 0$

$-4 = 0$ false

Solve and check

$\sqrt{x+6} + x = 6$

isolate radical first

$\sqrt{x+6} = 6 - x$

Square both sides

$\left(\sqrt{x+6}\right)^2 = (6-x)^2$

$x + 6 = (6-x)(6-x)$

Foil & Simplify

$x + 6 = 36 - 6x - 6x + x^2$

$x + 6 = 36 - 12x + x^2$

$36 - 12x + x^2 - x - 6 = 0$

$x^2 - 13x + 30 = 0$

$(x-10)(x-3) = 0$

$x - 10 = 0$ $\quad$ $x - 3 = 0$

$\cancel{x = 10}$ $\quad$ $\boxed{x = 3}$ ✓

E.S.

$\{3\}$

Check

$\sqrt{x+6} + x = 6$

$x = 10$	$x = 3$
$\sqrt{10+6} + 10 = 6$	$\sqrt{3+6} + 3 = 6$
$\sqrt{16} + 10 = 6$	$\sqrt{9} + 3 = 6$
$4 + 10 = 6$	$3 + 3 = 6$
$14 = 6$ false	$6 = 6$ ✓

Given $-2-5i$

1) Re. $=-2$ 2) Im. $=-5$ 3) Complex Conjugate $\boxed{-2+5i}$

Simplify and write in $a+bi$ form.

Re: $5\sqrt{2}$

Im. -7

$\sqrt{50}-\sqrt{-49}$

$=\sqrt{25}\sqrt{2}-\sqrt{49}\sqrt{-1}=5\sqrt{2}-7i$

$=\boxed{5\sqrt{2}-7i}$

Conjugate $5\sqrt{2}+7i$

Simplify

$2i(3-4i)-3(2+5i)$

$a+bi$

$=6i-8i^2-6-15i$

$=-9i-8(-1)-6=-9i+8-6=-9i+2$

$=\boxed{2-9i}$ Re. $=2$ Im. $=-9$

Simplify $(-2+3i)(4-5i)$

Use FOIL Method

$=-8+10i+12i-15i^2$

$a+bi$

$=-8+22i-15(-1)$

$=-8+22i+15$

$=\boxed{7+22i}$ Re. $=7$, Im. $=22$

Complex Conjugate

$7-22i$

Simplify $(3-4i)^2$

$= (3-4i)(3-4i)$

$= 9 - 12i - 12i + 16i^2$

$= 9 - 24i + 16(-1)$ $\quad a+bi$

$= 9 - 24i - 16 = \boxed{-7-24i}$

Re. $= -7$, Im. $= -24$

Complex Conjugate

$-7+24i$

Divide $\dfrac{5}{1-2i} = \dfrac{5(1+2i)}{(1-2i)(1+2i)}$

$= \dfrac{5+10i}{1 \cancel{+2i} \cancel{-2i} - 4i^2}$

$= \dfrac{5+10i}{1-4(-1)} = \dfrac{5+10i}{1+4}$

$= \dfrac{5+10i}{5}$

$= \dfrac{5}{5} + \dfrac{10}{5}i$

$\boxed{1+2i}$

Re. $= 1$ $\quad$ Im $= 2$

Divide $\dfrac{1+i}{2+3i} = \dfrac{(1+i)(2-3i)}{(2+3i)(2-3i)}$

$$= \frac{2-3i+2i-3i^2}{4\cancel{-6i}\cancel{+6i}-9i^2}$$

$$= \frac{2-i-3(-1)}{4-9(-1)} = \frac{2-i+3}{4+9}$$

$$= \frac{5-i}{13} = \boxed{\frac{5}{13}-\frac{1}{13}i}$$

$Re = \frac{5}{13}$

$Im. = \frac{-1}{13}$

Conjugate $\frac{5}{13}+\frac{1}{13}i$

Powers of i:

$(-1)^{odd} = -1$

Simplify $i^{250} = (i^2)^{125} = (-1)^{125} = \boxed{-1}$

$250 \to$ even

Simplify $i^{123} = i^{122} \cdot i = (i^2)^{61} \cdot i = (-1)^{61} \cdot i$

$123 \to$ odd

$122 \to$ even

$= \boxed{-i}$

$(-1)^{odd} = -1$ & $(-1)^{even} = 1$

Removing the radicals from numerator or denominator is called rationalizing.

Rationalize the deno.: $\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{10\sqrt{2}}{\sqrt{4}} = \frac{\cancel{10}^{5}\sqrt{2}}{\cancel{2}}$

$= \boxed{5\sqrt{2}}$

Rationalize the deno. : $\frac{x}{\sqrt{3x}}$

$\frac{x}{\sqrt{3x}} = \frac{x \cdot \sqrt{3x}}{\sqrt{3x} \cdot \sqrt{3x}} = \frac{x\sqrt{3x}}{\sqrt{9x^2}} = \frac{\cancel{x}\sqrt{3x}}{3\cancel{x}} = \boxed{\frac{\sqrt{3x}}{3}}$

Rationalize the deno.: $\frac{-2x}{\sqrt[3]{4x}} = \frac{-2x}{\sqrt[3]{2^2 x}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}}$

$= \frac{-2x\sqrt[3]{2x^2}}{\cancel{\sqrt[3]{2^3 x^3}}} = \frac{-\cancel{2x}\sqrt[3]{2x^2}}{\cancel{2x}}$

$= \boxed{-\sqrt[3]{2x^2}}$

Rationalize the deno.:

$$\frac{\sqrt{2}}{\sqrt{2}-1} = \frac{\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{\sqrt{4}+\sqrt{2}}{\sqrt{4}+\cancel{\sqrt{2}}-\cancel{\sqrt{2}}-1} = \frac{2+\sqrt{2}}{2-1} = \frac{2+\sqrt{2}}{1}$$

$$= \boxed{2+\sqrt{2}}$$

Rationalize the deno.:

$$\frac{7}{\sqrt{10}+\sqrt{3}} = \frac{7(\sqrt{10}-\sqrt{3})}{(\sqrt{10}+\sqrt{3})(\sqrt{10}-\sqrt{3})}$$

$$= \frac{7\sqrt{10}-7\sqrt{3}}{\sqrt{100}-\cancel{\sqrt{30}}+\cancel{\sqrt{30}}-\sqrt{9}}$$

$$= \frac{7\sqrt{10}-7\sqrt{3}}{10-3} = \frac{7\sqrt{10}-7\sqrt{3}}{7}$$

$$= \frac{7\sqrt{10}}{7} - \frac{7\sqrt{3}}{7}$$

$$= \boxed{\sqrt{10}-\sqrt{3}}$$

Rationalize the deno: $\dfrac{\sqrt{6}}{2\sqrt{3}-\sqrt{2}}$

$$\frac{\sqrt{6}\,(2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2})(2\sqrt{3}+\sqrt{2})} = \frac{2\sqrt{18}+\sqrt{12}}{4\sqrt{9}+\cancel{2\sqrt{6}}-\cancel{2\sqrt{6}}-\sqrt{4}}$$

$$= \frac{2\cdot\sqrt{9}\sqrt{2}+\sqrt{4}\sqrt{3}}{4\cdot 3-2}$$

$$= \frac{6\sqrt{2}-2\sqrt{3}}{10}$$

$$= \frac{\cancel{2}(3\sqrt{2}-\sqrt{3})}{\cancel{10}\,5}$$

$$= \boxed{\frac{3\sqrt{2}-\sqrt{3}}{5}}$$

Solve & check

$\sqrt{x+10} - 4 = x$

$\sqrt{x+10} = x + 4$

$(\sqrt{x+10})^2 = (x+4)^2$

$x + 10 = (x+4)(x+4)$

$x + 10 = x^2 + 4x + 4x + 16$

$x + 10 = x^2 + 8x + 16$

$x^2 + 8x + 16 - x - 10 = 0$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x + 6 = 0 \qquad x + 1 = 0$

$\cancel{x = -6} \qquad \boxed{x = -1}$

E.S.

$\{-1\}$

Check $x = -6$

$\sqrt{x+10} - 4 = x$

$\sqrt{-6+10} - 4 = -6$

$\sqrt{4} - 4 = -6$

$2 - 4 = -6$

$-2 = -6$ False

Check $x = -1$

$\sqrt{x+10} - 4 = x$

$\sqrt{-1+10} - 4 = -1$

$\sqrt{9} - 4 = -1$

$3 - 4 = -1$ ✓

Solve & Check

$\sqrt{x+7} + 5 = x$

$\sqrt{x+7} = x - 5$

$(\sqrt{x+7})^2 = (x-5)^2$

$x + 7 = x^2 - 10x + 25$

$x^2 - 10x + 25 - x - 7 = 0$

$x^2 - 11x + 18 = 0$

$(x-9)(x-2) = 0$

$x - 9 = 0 \qquad x - 2 = 0$

$\boxed{x = 9}$ ✓ $\qquad \cancel{x = 2}$

E.S.

$\{9\}$

Solve & check

$\sqrt[3]{(x+1)^2} - 4 = 0$

$\sqrt[3]{(x+1)^2} = 4$

Index $= 3$

$\left(\sqrt[3]{(x+1)^2}\right)^3 = (4)^3$

$(x+1)^2 = 64$

$x^2 + 2x + 1 - 64 = 0$

$x^2 + 2x - 63 = 0$

$(x+9)(x-7) = 0$

$x+9=0 \quad x-7=0$

$x=-9 \checkmark \quad x=7 \checkmark$

Check

$\{-9, 7\}$

Class QZ 15:

1) write in a+bi form: $\sqrt{18} - \sqrt{-100}$

2) Simplify: $-2i(5+i) + 5(3+2i)$

3) Simplify: $(3+4i)^2$

4) Divide: $\dfrac{13}{2-3i}$